OPTIMIZATION OF THERMOELECTRIC GENERATOR WITH SEGMENTED ELEMENTS

Drabkin I.A.[1], Ershova L.B.[2], Gochua K.V.[3]

[1] Institute of Chemical Problems for Microelectronics, 5 B. Tolmachevskiy per., Moscow, 109017, Russia
[2] RMT Ltd, 53 Leninskiy prosp., Moscow, 119991, Russia
[3] Moscow Institute of Steel and Alloys, 4 Leninskiy prosp., Moscow, 119049, Russia

Introduction

When designing thermoelectric (TE) generators it is an accepted practice to use segmented legs, i.e. legs comprised of consecutively attached TE materials for operation under various temperature intervals. This is due to the absence of thermal junctions resulting in smaller temperature losses in such legs compared to coupling of individual stages made of materials optimized for operation in respective temperature intervals. The segment size of such a leg and the generator current is picked on the basis of maximizing generator COP at given temperatures of hot and cold ends of the leg. This is a problem belonging to those of optimization control, and it is convenient to solve it using the Pontryagin Maximum Principle [1]. This method was successfully used for optimization of TE coolers [2] and multistage TE generators. [3]. This paper gives a method for calculation of optimal segment size of the generator leg and expressions for optimal currents.

Optimizing Operation of the Generator Thermoelement with Segmented Legs

Let us consider a generator thermoelement with legs composed of various thermoelectric materials. The layout for such thermoelement is shown in Fig. 1.

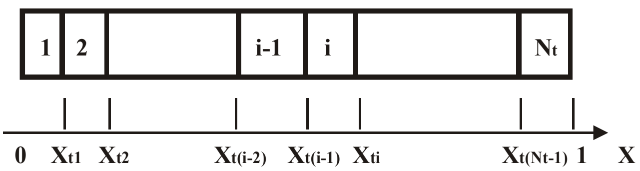

Fig. 1 Layout of the segmented thermoelement leg

Let the p-type leg consist of N_p segments, and the n-type leg consist of N_n segments, respectively. The value of N_p can differ from that of N_n. We will be marking parameters and variables related to a segment with two indices: t is always attributed to the conductivity type, and i refers to the number of the segment. The Seebeck coefficient is α, electrical conductivity is σ, resistivity is ρ, and thermal conductivity is κ. The temperatures attributed to the segment ends with coordinate X_{ti} are marked as T_{ti}, and the temperature limit to the right of X_{ti} is denoted as $T(x_{ti}+0)$, and to the left is denoted as $T(x_{ti}-0)$. The heat conduction equation for i-segment of the leg is:

$$\nabla(\kappa_{ti}\nabla T_{ti})+(\vec{j},\vec{j})\rho_{ti}-T_{ti}(\vec{j},\nabla\alpha_{ti})=0 \quad (1)$$

where $t=n,p; i=1,2,..N_t$, $\vec{j}$ is the current density through the sample. Further on we will consider the problem in one dimension. This way it is possible to turn one-dimensional vectors into scalars, respectively changing the signs for these values in Eq. (1) and considering the current and thermal flux to be always positive. As the current direction is different in the legs of n- and p-types, and the product of αj retains the sign, j is positive, the Seebeck coefficient will be $|\alpha|$. The equations system (1) for this case is as follows:

$$\begin{cases} \frac{\partial T_{ti}}{\partial x} = -\frac{j\alpha_{ti}T_{ti}}{\kappa_{ti}} + \frac{q_{ti}j}{\kappa_{ti}} \\ \frac{\partial q}{\partial x} = -\frac{j\alpha_{ti}^2}{\kappa_{ti}}T_{ti} + \frac{j\alpha_{ti}}{\kappa_{ti}}q_{ti} - j\rho_{ti} \end{cases}, \qquad (2)$$

where

$$q_{ti} = \frac{\left(\kappa_{ti}\frac{\partial T_{ti}}{\partial x} + T_{ti}j\alpha_{ti}\right)}{j} \qquad (3)$$

is the thermal flux density divided by the current density.

System (2) is solved under the following boundary conditions: the temperature of the cold leg end $T_{t1}(0) = T_c$, the temperature of the hot leg end $T_{tN_t}(1) = T_h$, on the segments connection $T_{t(i-1)}(x_{t(i-1)} - 0) = T_{ti}(x_{t(i-1)} + 0) - \delta_{ti}$, where symbol δ_{ti} denotes temperature losses commutation between the segments. Besides, $q_{t(i-1)}(x_{t(i-1)} - 0) = q_{ti}(x_{t(i-1)} + 0)$. This problem is reduced to the Cauchy problem and is solved using standard approaches.

Hamiltonian [1] for equations system (2) is a sum of partial Hamiltonians for the leg segments:

$$H = \sum_{\substack{t=n,p \\ i_t=1,..N_t}} H_{ti_t} = \sum_{\substack{t=n,p \\ i_t=1,..N}} \left[\psi_{1i_t}\left(\frac{j}{\kappa_{ti_t}}q_{ti_t} - \frac{j\alpha_{ti_t}T_t}{\kappa_{ti_t}}\right) + \right.$$
$$\left. + \psi_{2i_t}\left(-\frac{j}{\sigma_{ti_t}} + \alpha_{ti_t}\frac{j}{\kappa_{ti_t}}q_{ti_t} - \frac{j(\alpha_{ti_t})^2 T_t}{\kappa_{\alpha_{ti_t}}}\right)\right] \qquad (4)$$

where ψ_{1i_t} and ψ_{2i_t} are variables, conjugated [1] respectively to T and q. The conjugated equations system for variables ψ_{1i_t} and ψ_{2i_t} is

$$\begin{cases} \frac{\partial \psi_{1i_t}}{\partial t} = \psi_{1i_t}\frac{j\alpha_{i_t}}{\kappa_{i_t}}R_{1i_t} + \psi_{2i_t}\frac{j\alpha_{i_t}^2}{\kappa_{i_t}}R_{2i_t} \\ \frac{\partial \psi_{2i_t}}{\partial t} = \frac{j}{\kappa_{i_t}}\psi_{1i_t} - \frac{j\alpha_{i_t}}{\kappa_{i_t}}\psi_{2i_t} \end{cases} \qquad (5)$$

where, in accordance with [2]:

$$R_{1i_t} = \left(1 + T_t\frac{d\ln\alpha_{i_t}}{dT_t} - \frac{d\ln\kappa_{i_t}}{dT_t}\left(T_t - \frac{q_{i_t}}{\alpha_{i_t}}\right)\right) \qquad (6)$$

$$R_{2i_t} = R_{1i_t} + \frac{d\ln\alpha_{i_t}}{dT_t}\left(T_t - \frac{q_{i_t}}{\alpha_{i_t}}\right) - \frac{1}{Z_{i_t}}\frac{d\ln\sigma_{i_t}}{dT_t} \qquad (7)$$

$$Z_{i_t} = \frac{\alpha_{i_t}^2\sigma_{i_t}}{\kappa_{i_t}} \qquad (8)$$

In the optimal mode to maximize COP each thermoelement leg shall provide maximum COP. Moreover each segment also shall operate at maximum COP. That is why the condition for optimal solution is the minimum of the functional

$$J = \sum_{t=n,p} ln\left(\frac{B}{C}\right) \qquad (9)$$

$$B = q_{tN_t}(1-0)q_{t1}(x_1 - 0)q_{t2}(x_2 - 0)..q_{tN_t}(x_{t(N_t-1)} - 0)$$
$$C = q_{t1}(0+0)q_{t2}(x_2 + 0)q_{t3}(x_3 + 0)..q_{tN_t}(x_{t(N_t-1)} + 0)$$

Considering the system connections, the generalized functional $\bar{J}$ is:

$$\bar{J} = J + \sum_{t=n,p}\lambda_{1t}(T_{1t}(0+0) - T_c) + \sum_{t=n,p}\lambda_{2t}(T_{N_t}(1-0) - T_h) +$$
$$+ \sum_{\substack{t=n,p \\ i_t=1,..N_t}}\left[\lambda_{3t_i}\left(T_{t(i-1)}(x_{t(i-1)} - 0) - T_{ti}(x_{t(i-1)} + 0) + \delta_{ti}\right)\right] \qquad (10)$$

The transversal conditions [2] give the boundary conditions for the solution (5):

$$\psi_{2t}(x_{ti}) = -\frac{1}{q_{ti}(x_{ti})}, t = n, p,\ i = 1,..,N_t \qquad (11a)$$

$$\psi_{1t(i-1)}(x_{t(i-1)} - 0) = \psi_{1t_i}(x_{t(i-1)} + 0) \qquad (11b)$$

$$t = p, i = 1,2,..N_p;\ t = n, i = 1,2,..N_n$$

The optimal current j_{opt} is

$$j_{opt} = \frac{1}{A}\sum_{\substack{t=n,p \\ i_t=1,..N_t}}\psi_{2t_i}\alpha_{t_i}T_i\Bigg|_{(x_{ini})_{ti}}^{(x_{end})_{ti}} \qquad (12)$$

$$A = \sum_{\substack{t=n,p \\ i_t=1,..N_t}}\int_{(x_{ini})_{ti}}^{(x_{end})_{ti}}\left[\left(\frac{\psi_{2t_i}}{\sigma_{t_i}}(1 - Z_{t_i}T_t) - \frac{q_{t_i}\psi_{1t_i}}{\kappa_{t_i}}\right) + \right.$$
$$\left. + \frac{\psi_{2t_i}T_t}{\kappa_{t_i}}\frac{d\alpha_{t_i}}{dT_t}(q_{t_i} - \alpha_{t_i}T_t)\right]dx$$

where the symbols $(x_{ini})_{ti}$ and $(x_{end})_{ti}$ denote the beginning and the end of the corresponding segment. When $N_p = N_n = 1$ and the thermoelectric parameters do not depend on the temperature, Eq. (12) yields the known expression $j_{opt} = \frac{\alpha\sigma(T_h - T_c)}{1+\sqrt{1+Z(T_h - T_c)/2}}$ [4].

The algorithm for finding the optimal segment length is the following: we take any distribution of the segment lengths; at zero approximation we consider the distribution of temperature along the leg to be linear and substitute thermoelectric coefficients with their average temperature values; after that we calculate the values of $j^{(0)}$ and $q_{t_1}(0)^{(0)}$, $t = n, p$. Then we solve system (2) by changing $q_{t_1}(0)$, achieving boundary conditions $T_{t_{N_t}} = T_h, t = n, p$. After determining the distribution of temperatures we use (11) to determine boundary conditions for $\psi_{2t_i}, t = n, p;\ i = 1,2..N_t$, solving system (5), and determine values $\psi_{1t_i}, t = n, p;\ i = 1,2..N_t$. We calculate the current at the first approximation $j^{(1)}$ using (12). As the values $\psi_{1t_i}, t = n, p;\ i = 1,2..N_t$ do not satisfy condition (11), we change the values $x_{t_i}, t = n, p;\ i = 1,2..(N-1)_t$ to satisfy (11), approximating dependency $\psi_{1t_i}(x)$, at least linearly, we find the distribution of segment lengths at the first approximation. After that we take the new value for the current and new distribution of segment lengths, solve equation (2) and repeat the procedure described above until the changes in thermoelement COP stop exceeding the limits of the set accuracy.

TE generator EMF value E is calculated as:

$$E = \sum_{\substack{t=n,p \\ i=1,..,N_t}} \int_{x_{t(i-1)}}^{x_{ti}} \alpha(T_t) \frac{dT_i}{dx_i} dx_i \qquad (13)$$

And the internal resistance R equals

$$R = \sum_{\substack{t=n,p \\ i=1,..,N_t}} \int_{x_{t(i-1)}}^{x_{ti}} \rho(T_t) \frac{dx_i}{dT_i} dT_i \qquad (14)$$

If cross-sections of the segments are arbitrary, the solution is simpler as there is no need to set the distribution of the segment lengths. The problem can be solved for the segment lengths equaling one, and after that the necessary lengths are picked out, the geometry factor being used.

Conclusion

This paper gives a method for calculation of the optimal segment size of the TE generator segmented leg and expressions for the optimal currents, the method of Pontriagin maximum control being applied. The results are applicable for the engineering mathematical simulation of TE generators.

Literature

1. Pontriagin L.S., Boltiansky V.G., Gamkrelidze Z.I., Mischenko E.F. Mathematical theory of Optimal Processes, (Moscow,1983), pp.13-132.

2. Anatychuk L.I., Semeniuk V.A. Optimal Control of Thermoelectric Properties and Devices, Prut (Chernivtsi, 1992), pp. 68-96

3. Anatychuk L.I., Vikhor L.N. Computer design of thermoelectric generator modules. Thermoelectricity (2005) ,N 2, pp. 62-70.

4. Ohotin F.B., Efremov A.A., Ohotin V.S., Pushkarsky A.S. Thermoelectric generators, (Moscow, Atomizdat, 1971), pp. 3-318.